Universal thermodynamics and two-body contact of a Bose gas near the superfluid–Mott-insulator transition

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The two-body contact is a fundamental quantity of a dilute Bose gas that relates the thermodynamics to the short-distance two-body correlations. For a Bose gas in an optical lattice, near the superfluid–Mott-insulator transition, we show that a "universal" contact $C_{\rm univ}$ can be defined from the singular part $P - P_{\rm MI}$ of the pressure ($P_{\rm MI}$ is the pressure of the Mott insulator). Its expression $C_{\rm univ} = C_{\rm DBG}(|n - n_{\rm MI}|, a^*)$ coincides with that of a dilute Bose gas provided we consider the effective "scattering length" a^* of the quasi-particles at the quantum critical point (QCP) rather than the scattering length in vacuum, and the excess density $|n - n_{\rm MI}|$ of particles (or holes) with respect to the Mott insulator. Close to the transition, we find that the singular part $n^{\rm sing}_{\mathbf{k}} = n_{\mathbf{k}} - n^{\rm MI}_{\mathbf{k}}$ of the momentum distribution exhibits a high-momentum tail of the form $Z_{\rm QP} C_{\rm univ}/|\mathbf{k}|^4$ over a broad region of the Brillouin zone, where $Z_{\rm QP}$ is the quasi-particle weight of the elementary excitations at the QCP. Our results demonstrate that the notion of contact extends to strongly correlated lattice bosons.

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[1] M. Bhateja, N. Dupuis, and A. Rançon, Two-body contact of a Bose gas near the superfluid–Mott-insulator transition, Phys. Rev. A **112**, L041301 (2025).